This is an overview of ECE314 Lab 2.

This lab has three parts. First, we will talk about histogram. Second, the Law of Large Numbers. This is the key concept in this lab. Third, we will see how the law of large numbers can be applied in the simulation of some simple games, including the poker hands and the monty hall problem.

First, histogram. Histogram is a graphical representation of simulation results. It shows the frequency at which values occur. For example, suppose you toss a dice 1000 times and you get 220 ones, 180 twos, 100 threes, 150 fours, 150 fives and 200 sixes. We can plot the histogram of this experiment as below. In the figure, there are six bars, the height of the each bar represents the frequency of each value.

In the lab, you will learn how to plot a histogram in Python.

Next, let us talk about the law of large numbers. The law of large numbers is a theory. The name of theory sounds technical, but idea it conveys is simple.

Consider this question: if you toss a fair dice one million times and calculate the average value, if there is anything we can say about this value, what do you expect this value to be like?

One answer could be: it is close to 3.5. Hopefully that makes sense to you. If you also get this answer, you have been using the law of large number implicitly in your mind.

More formally, the law of large numbers says the following. Let X1, X2, dots, Xn be n independent random variables with the same distribution. You can think of it as the results of doing the same experiment n times. Let mu be the mean of the distribution. Then the average of the n values will converge to mu in some proper sense.

One particular proper sense is given mathematically in the lab and in the course notes of ECE313 in a later chapter.

Now, let us see how the law of large number is used in simulations.

First, consider randomly drawing a hand of five cards from a deck of 52 cards. We ask the question: what is the probably of getting a flush?

Using what you have learned in ECE313, you can provide an exact solution to this question. Now, suppose you have not learned ECE313 and you don’t have the math knowledge to answer this question exactly, how can you try to answer this question?

One obvious idea is this: you get a real deck of cards and draw 5 cards randomly, check if is a flush. Then you do it again, and check. You do it many times, say a million times. At the end, you count the total number of times you got a flush, then you divide that number by one million. You then obtain a probability. You expect that number to be close to the exact answer. Why?

The reason behind this idea is the law of large numbers. If you get a flush, it is a success, a success is a one; if you don’t get a flush, it is a failure, a failure is a zero. The probability of flush would be mean of a Bernoullil random variable, which is unknown to you. By the law of large numbers, when you repeat the experiment by enough number of times, the average of the ones and zeros should be close to the mean.

You will apply this idea in almost all the simulations in the labs throughout this course.

In the lab, you will write a program to simulate the process of drawing cards. You will observe and verify how close your empirical result is to the theoretical result.

Finally, let us consider the interesting Monty Hall Problem, as follows.

You go to a show. The host invites you to play a game. There are three doors. Behind one door there is a goat, behind the other two doors there is nothing. You don’t know which door has the goat.

Now, you choose one door. Any door.

For the purpose of illustration, suppose you choose this door.

Now, the host, who knows which door has the goat, reveals one empty door to you. Note that it is always possible for the host to do so, because no matter which door you choose, there is always an empty door in the remaining two doors.

Suppose this is the empty door revealed to you.

Now, you have two choices: 1. Stick with your original choice. 2. Switch to the other door remained.

The question is: should you switch?

Asked in another way: what is the probability of winning the goat if you switch and what is the probability of winning the goat if you do not switch?

Hopefully you can spend some time to think about this problem. Then, you can write a program in the lab to simulate this game and to verify your thoughts.

That concludes the overview of this lab.